1 Production

Profit (π) The difference between revenues (R) and costs (C).

$$\pi = R - C$$

Capital (K) Inputs that are not exhausted with a single use.

Labor (L) Human inputs into production.

Materials Raw goods that are used as inputs to production.

Production Function A function that gives the maximum amount of output we can make given the quantities of inputs that we use.

$$q = f(L, K)$$

Short Run A period of time such that at least one production input is fixed.

Fixed Input An input that cannot be changed in the short run. This is usually capital.

Variable Input An input that can be changed even if we are in the short run. This is usually labor.

Short Run Production Function A production function where capital is fixed so only labor can vary.

$$q = f(L, \bar{K})$$

Long Run A period of time such that all production inputs can be changed.

Total Product of Labor The total amount of output that can be produced by a given amount of labor.

Marginal Product of Labor The amount of output than an additional worker will provide, keeping all other factors constant.

$$MP_L = \frac{\partial q}{\partial L} = \frac{\partial f(L, \bar{K})}{\partial L}$$

Average Product of Labor The average amount of output produced by each worker.

$$AP_L = \frac{q}{L} = \frac{f(L, \bar{K})}{L}$$

Law of Diminishing Marginal Returns As we increase the use of a certain input, the marginal productivity of that input will eventually decrease if all other inputs are kept constant.

Isoquant A curve that shows the different combinations of labor and capital that produce a single level of output.

$$\bar{q} = f(L, K)$$

Properties of Isoquants:

1. Isoquants represent larger quantities as you move in the northeast direction.
2. Isoquants can never cross.
3. Isoquants slope downwards.

Marginal Rate of Technical Substitution The number of extra units of capital needed to replace one unit of labor that enables a firm to produce the same level of output. Essentially how many units of capital a worker is equivalent to.

$$MRTS = -\frac{\frac{\partial f(L,K)}{\partial L}}{\frac{\partial f(L,K)}{\partial K}} \approx \frac{\Delta K}{\Delta L}$$

Constant Returns to Scale A production function such that if we increase every input by a certain percentage, then output also increases by that same percentage.

$$f(\lambda L, \lambda K) = \lambda f(L, K)$$

Increasing Returns to Scale A production function such that if we increase every input by a certain percentage, then output will increase by *more* than that percentage.

$$f(\lambda L, \lambda K) > \lambda f(L, K)$$

Decreasing Returns to Scale A production function such that if we increase every input by a certain percentage, then output will increase by *less* than that percentage.

$$f(\lambda L, \lambda K) < \lambda f(L, K)$$

2 Input Choices and Costs

Isocost Line All the combinations of labor and capital that have to same total cost.

$$\bar{C} = wL + rK$$

At optimal point (L^*, K^*) where a firm is producing a given quantity at the lowest possible cost, the marginal rate of technical substitution is equal to the input price ratio. At this point the output per dollar for labor and capital are equal.

$$MRTS = -\frac{w}{r} \Leftrightarrow -\frac{MP_L}{MP_K} = -\frac{w}{r} \Leftrightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$$

Expansion Path The quantities of labor and capital that minimize cost for any given output level.

3 Exercises

1. Steve is running a company that makes extra strength sunscreen for gingers. The quantity of sunscreen Q_S Steve can make from labor and capital is given by the production function $Q_S(L, K) = L^{\frac{1}{4}} K^{\frac{3}{4}}$.

   (a) Does Steve's firm display constant, increasing, or decreasing returns to scale?

   $$Q_S(\lambda L, \lambda K) = (\lambda L)^{\frac{1}{4}} (\lambda K)^{\frac{3}{4}} = \lambda^{\frac{1}{4}} L^{\frac{1}{4}} \lambda^{\frac{3}{4}} K^{\frac{3}{4}} = \lambda L^{\frac{1}{4}} K^{\frac{3}{4}} = \lambda Q_S(L, K)$$

   Thus Steve's firm has constant returns to scale. Because Steve's company has a Cobb-Douglas production function, we can also tell there are constant returns to scale because the sum of the exponents is equal to 1. For Cobb-Douglas, if the sum of the exponents are greater than 1 there are increasing returns to scale, and if the sum of the exponents are less than 1 there are decreasing returns to scale.

   (b) Suppose the current wage is \$8 and the current rental rate is \$48. Steve has decided to use 80 units of labor and 30 units of capital to produce his current level of output. Is he minimizing the costs for his firm? If not, should he use more labor or more capital?
   At the cost minimizing level of labor and capital, the MRTS is equal to the negative ratio of input prices.

   $$MRTS = -\frac{w}{r} \Leftrightarrow -\frac{\frac{\partial Q_S(L,K)}{\partial L}}{\frac{\partial Q_S(L,K)}{\partial K}} = -\frac{w}{r} \Leftrightarrow \frac{\frac{1}{4} L^{-\frac{3}{4}} K^{\frac{3}{4}}}{\frac{3}{4} L^{\frac{1}{4}} K^{-\frac{1}{4}}} = \frac{8}{48} \Leftrightarrow \frac{1}{3}\frac{K}{L} = \frac{1}{6} \Leftrightarrow L = 2K$$

   So Steve's firm should be using twice as much labor as capital. However, his firm's ratio of labor to capital is $\frac{80}{30} > 2$. Thus Steve's firm is using too much labor and should replace some of the workers with capital.

(c) Can Steve's firm produce the same quantity and get to the cost minimizing level of labor and capital in the short run?

Because capital is fixed in the short run, Steve's firm is stuck with a suboptimal level of labor and capital for now. They will be able to hire less labor in the short-run, but will only be able to rent more capital in the long-run.

2. Jalen has invented a new type of basketball shoes with springs that allows short people to dunk. His cost function of producing the shoes is given by $C(q) = 4q^2 - 20q + 100$. Suppose the \$100 fixed cost is not *sunk*, so that Jalen does not have to pay it if he decides not to produce.

(a) What is the average cost for producing shoes?

$$AC = \frac{C(q)}{q} = \frac{4q^2 - 20q + 100}{q} = 4q - 20 + \frac{100}{q}$$

(b) What is the marginal cost for producing shoes?

$$MC = C'(q) = 8q - 20$$

(c) When is average cost increasing? The average cost is increasing whenever the marginal cost is greater than the average cost.

$$MC > AC \Leftrightarrow 8q - 20 > 4q - 20 + \frac{100}{q} \Leftrightarrow 4q > \frac{100}{q} \Leftrightarrow 4q^2 > 100 \Leftrightarrow q > 5$$

(d) What is the average variable cost?

$$AVC = \frac{4q^2 - 20q}{q} = 4q - 20$$

(e) If the price that Jalen can sell shoes at is \$12, how much will Jalen produce? What is his profit. The firm will produce until the price is equal to marginal cost.

$$p = MC \Leftrightarrow 12 = 8q - 20 \Leftrightarrow q^* = 4$$

To see if Jalen makes positive profits at this quantity, we need to see if the price (average revenue) is greater than the average cost.

$$AC = 4(4) - 20 + \frac{100}{4} = 21 > 12 = p$$

Thus the average cost is greater than the price, so Jalen will make a negative profit by producing 4 pairs of shoes. Because his fixed cost is not sunk, it is best for Jalen to produce no shoes and make zero profits, as opposed to making negative profits by making 4 pairs of shoes.

$$q^* = 0 \quad \pi = 0$$

(f) If the price that Jalen can sell shoes at is \$28, how much will Jalen produce? What is his profit? Again, the optimal quantity occurs where price is equal to marginal cost.

$$p = MC \Leftrightarrow 28 = 8q - 20 \Leftrightarrow q^* = 6$$

Now let's see if Jalen makes positive profits.

$$AC = 4(6) - 20 + \frac{100}{6} = 20.67 < 28 = p$$

Thus Jalen's cost per pair of shoes is less than he takes in by selling a pair of shoes, so he will make a positive profit and it makes sense to produce the 6 pairs of shoes. We can find his profit by subtracting costs from revenues.

$$\pi = p \cdot 6 - C(6) = 28 \cdot 6 - (4 \cdot 6^2 - 20 \cdot 6 + 100) = 168 - 124 = \$44$$