

1 Budget Sets

Budget Line The bundles of goods we can purchase if we spend all of our money.

$$P_x x + P_y y = W$$

Opportunity Set The bundles of goods we can afford, even if we don't spend all of our money.

Marginal Rate of Transformation How much of good y you must give up in order to purchase an additional unit of good x in the marketplace. This is equal to the slope of the budget line.

$$MRT = -\frac{P_x}{P_y}$$

If the price of one of the goods changes, then the *slope* of the budget constraint will change. If the wealth of a consumer changes, then the slope remains the same but the budget line *shifts*.

2 Constrained Consumer Optimization

Interior Solution An optimal bundle where the consumer's MRS is equal to the MRT.

Corner Solution An optimal bundle where the consumer's MRS is *not* equal to the MRT. This will generally happen if the consumer is only consuming one good but would still like to consume more of that good, or if the budget constraint has a kink/corner.

For an *interior solution*, the optimal bundle occurs when our personal tradeoff (MRS) is equal to the market tradeoff (MRT), and we are spending all of our wealth. At this point our utils per dollar are the same for both goods, and we have no income to purchase any more goods.

$$MRS = MRT \Leftrightarrow -\frac{MU_x}{MU_y} = -\frac{P_x}{P_y} \Leftrightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

3 Exercises

- Yuzuru's utility is a function of chocolate bars, c , and sake, s , which can be represented by $U(c, s) = c^{0.3}s^{0.7}$. His wealth is \$400. The price of chocolate bars is \$4, and the price of sake is \$10.

- What is Yuzuru's marginal rate of substitution? What is the marginal rate of transformation?

$$MRS = -\frac{\frac{\partial U}{\partial c}}{\frac{\partial U}{\partial s}} = -\frac{0.3c^{-0.7}s^{0.7}}{0.7c^{0.3}s^{-0.3}} = -\frac{3s}{7c} \quad MRT = -\frac{P_c}{P_s} = -\frac{4}{10} = -\frac{2}{5}$$

- How many chocolate bars and ounces of sake will Yuzuru decide to consume?

At Yuzuru's optimal bundle, $MRS = MRT$.

$$-\frac{3s}{7c} = -\frac{2}{5} \Leftrightarrow s = \frac{14}{15}c$$

We also know that Yuzuru will use all of his wealth and choose a bundle on his budget constraint.

$$4c + 10s = 400 \Leftrightarrow s = \frac{400 - 4c}{10}$$

Combining the two gives the optimal bundle.

$$\frac{14}{15}c = \frac{400 - 4c}{10} \Leftrightarrow c = 30 \quad s = \frac{14}{15} \cdot 30 \Leftrightarrow s = 28$$

- (c) Suppose the price of sake drops to \$5. Does Yuzuru consume more or less sake? Does he consume more or less chocolate bars?

Applying the same steps gives $c = 30$ and $s = 56$. Notice that the quantity of chocolate bars has not changed. This is a unique property of Cobb-Douglas preferences.

2. Cody's utility is a function of cheese, c , and cat videos, v , which can be represented by $U(c, v) = 2c + v$. His wealth is \$200.

- (a) How much cheese will Cody eat and how many cat videos will he watch if the price of cheese is \$8 and the price of cat videos is \$4.25?

$$MRS = -\frac{2}{1} = -2 \quad MRT = -\frac{8}{4.25} \approx -1.882$$

Cody is willing to give up two cat videos for one block of cheese, but he has to give up less than two videos per block of cheese in the market. So Cody will consume only cheese and $c = 25$ and $v = 0$.

- (b) How much cheese will Cody eat and how many cat videos will he watch if the price of cheese is \$8.25 and the price of cat videos is \$4?

$$MRS = -\frac{2}{1} = -2 \quad MRT = -\frac{8.25}{4} = -2.0625$$

Cody is willing to give up two cat videos for one block of cheese, but he has to give up more than two videos per block of cheese in the market. So Cody will consume only cat videos and $c = 0$ and $v = 50$.

- (c) How much cheese will Cody eat and how many cat videos will he watch if the price of cheese is \$8 and the price of cat videos is \$4?

$$MRS = -\frac{2}{1} = -2 \quad MRT = -\frac{8}{4} = -2$$

Cody is willing to give up two cat videos for one block of cheese, which is exactly the market tradeoff. Therefore he will be happy as long as he spends all of his money, so any bundle along his budget constraint will work.